Einstein Equations, Dark Matter and Dark Energy

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I. Einstein's General Relativity

Inspired by the vision of Albert Einstein and Paul Dirac, ..., our study follows the two guiding principles; see e.g. [Ma-Wang, Mathematical Principles of Theoretical Physics, Science Press, 2015, pp. 550]:

Guiding Principle 1

1. the entire theoretical physics is built upon a few fundamental first principles;
2. the laws of Nature are simple and aesthetic with clear physical pictures.

Guiding Principle 2

All physical systems obey laws and principles of Nature. For each system,

1. there is a group of functions $u = (u_1, \cdots, u_N)$ describing its states, and the laws and principles obeyed by the system can be expressed as:

$$\text{physical laws} \ = \ \text{mathematical equations};$$

2. there is a functional $F(u)$, which dictates mathematical equations, and usually represents a specific form of energy;
3. All physical systems obey certain symmetries, which essentially determine the mathematical forms of the functionals F.

General relativity (Albert Einstein, 1915)

Principle of Equivalence (PE): the space-time is a 4D Riemannian manifold $(M, g_{\mu\nu})$ with $g_{\mu\nu}$ being regarded as gravitational potentials.

Principle of General Relativity (PGR): the law of gravity be covariant under general coordinate transformations, and dictates the Einstein-Hilbert action:

$$L_{EH}(\{g_{\mu\nu}\}) = \int_M \left(R + \frac{8\pi G}{c^4} S \right) \sqrt{-g} dx. \tag{1}$$

Euler-Lagrangian equations of L_{EH}:

$$0 = \frac{d}{d\lambda}\Big|_{\lambda=0} L_{EH}(g_{\mu\nu} + \lambda X_{\mu\nu}) = (\delta L_{EH}(g_{\mu\nu}), X_{\mu\nu}) \qquad \forall X_{\mu\nu}.$$

Einstein gravitational field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}, \qquad \nabla^\mu T_{\mu\nu} = 0. \tag{2}$$

Schwarzschild solution (1916)

Consider a centrally symmetric gravitational field generated by a ball B_R with radius R and mass m. **Outside the ball ($r > R$)**, the metric takes the form:

$$ds^2 = -e^u c^2 dt^2 + e^v dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad u = u(r), v = v(r). \tag{3}$$

Then the Einstein equations take the form

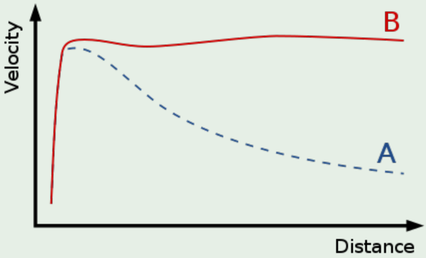
$$\begin{aligned} & v' + \frac{1}{r}(e^v - 1) = 0, \\ & u' - \frac{1}{r}(e^v - 1) = 0, \\ & u'' + \left(\frac{1}{2}u' + \frac{1}{r}\right)(u' - v') = 0. \end{aligned} \tag{4}$$

Schwarzschild solution of the Einstein equations ($R_s = 2mG/c^2$):

$$ds^2 = -\left(1 - \frac{R_s}{r}\right)c^2 dt^2 + \left(1 - \frac{R_s}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2. \tag{5}$$

II. New Gravitational Field Equations (Ma-Wang, 2012)

Dark matter and dark energy



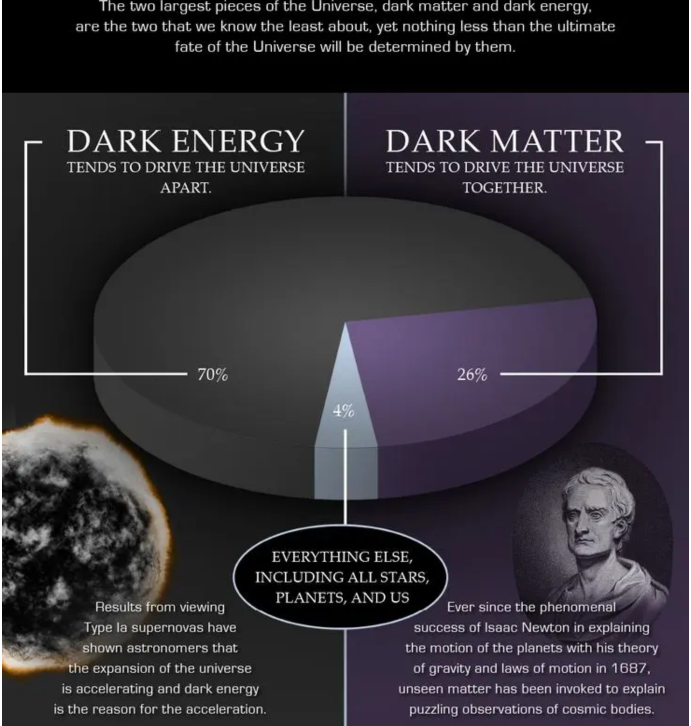
Galactic rotation curve of a typical spiral galaxy: predicted (A) and observed (B).

$$\frac{GM(r)}{r^2} = \frac{v^2}{r} \iff v^2 = \frac{GM(r)}{r}$$

The notion of dark matter was introduced to explain the 'flat' appearance of the velocity curve out to a large radius.

People are searching for potential candidates of dark matter, including e.g. weakly interacting massive particles (WIMP). In more than 50 years, many people are involved in detecting dark matter such as Alpha Magnetic Spectrometer (AMS) (Samuel Ding, ...). So far there are still no trace of it!

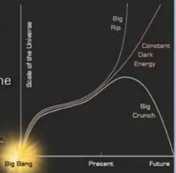
In the late 1990s, astronomers found evidence that the expansion of the universe was increasing. **Dark energy** is the name given to the mysterious force that's causing the rate of expansion of our universe to accelerate over time.


The two largest pieces of the Universe, dark matter and dark energy, are the two that we know the least about, yet nothing less than the ultimate fate of the Universe will be determined by them.
DARK ENERGY
TENDS TO DRIVE THE UNIVERSE APART.
DARK MATTER
TENDS TO DRIVE THE UNIVERSE TOGETHER.
70%
26%
4%
EVERYTHING ELSE, INCLUDING ALL STARS, PLANETS, AND US
Results from viewing Type Ia supernovas have shown astronomers that the expansion of the universe is accelerating and dark energy is the reason for the acceleration.
Ever since the phenomenal success of Isaac Newton in explaining the motion of the planets with his theory of gravity and laws of motion in 1687, unseen matter has been invoked to explain puzzling observations of cosmic bodies.

The two basic models for dark energy are that it is either energy associated with empty space (vacuum energy) and is constant throughout space and time, or it is an energy field that varies over space and time.

If vacuum energy is correct in about 100 billion years, no galaxy outside our own will be visible.

If it is an energy field, depending on the nature, expansion could stop and become a collapse, or the acceleration could increase without limit.



If dark energy does not exist, cosmic acceleration could be a sign that the theory of gravity needs to be modified for extremely large distance scales.

Evidence suggest that the mass of dark matter in galaxies, clusters of galaxies, and the universe as a whole is about 5 or 6 times greater than the mass of ordinary baryonic matter such as protons and neutrons.

Dark matter is thought to be mostly composed of exotic particles formed when the universe was a fraction of a second old.

The issue of whether dark matter exists or gravity needs to be modified will likely not be resolved until dark matter particles are detected, or ruled out by lack of detection.

Some physicists propose actually making dark matter.

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New Gravitational Field Equations (Ma-Wang, 2012)

The presence of dark matter and dark energy implies that the energy-momentum tensor of visible matter $T_{\mu\nu}$ may no longer be conserved: $\nabla^\mu T_{\mu\nu} \neq 0$.

Consequently, the variation of L_{EH} must be taken under energy-momentum conservation constraint, leading us to postulate a general principle, called principle of interaction dynamics (PID) (Ma-Wang, 2012):

$$\frac{d}{d\lambda}\Big|_{\lambda=0} L_{EH}(g_{\mu\nu} + \lambda X_{\mu\nu}) = 0 \quad \forall X = \{X_{\mu\nu}\} \text{ with } \nabla^\mu X_{\mu\nu} = 0. \tag{6}$$

Then we derive a new set of gravitational field equations

$$\begin{aligned} & R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu, \\ & \nabla^\mu \left[\frac{8\pi G}{c^4} T_{\mu\nu} + \nabla_\mu \Phi_\nu \right] = 0. \end{aligned} \tag{7}$$

Remarks

- The Einstein-Hilbert action is dictated by the symmetry principle (principle of general relativity), together with simplicity of laws of Nature, and the Einstein-Hilbert action should **not** be altered.
- The new term $\nabla_\mu \Phi_\nu$ is non-variational, and cannot be derived
  - from any existing $f(R)$ theories, and
  - from any scalar/tensor field theories.

  Namely,

  > *the term $\nabla_\mu \Phi_\nu$ does not correspond to any Lagrangian action density, and is the direct consequence of PID.*

- The field equations (7) establish a natural **duality**:

  gravitational field $\{g_{\mu\nu}\}$ $\longleftrightarrow$ dual vector gravitational field $\{\Phi_\mu\}$

Non Well-Posedness of the Einstein Equations

Landau: The four coordinates x^i can be subjected to an arbitrary transformation. By means of these transformations we can arbitrarily assign four of the ten of the 10 components of the tensor g_{ik}. Therefore there are only six independent quantities g_{ik}.

The Einstein equations contain 10 equations with 6 unknowns $\{g_{\mu\nu}\}$, and may lead to ill-posedness in general. With the introduction of Ψ_μ, the new gravitational field equations contain exactly 10 equations, resolving this difficulty.

Example: The metric of central gravitational field takes the form

$$ds^2 = -c^2 e^u dt^2 + e^v dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad u = u(r,t), v = v(r,t).$$

Take $T_{\mu\nu} = \begin{pmatrix} -g_{00}\rho & 0 \\ 0 & 0 \end{pmatrix}$, where ρ is the energy density, a constant. Then the Einstein equations have no solutions.

In fact, the Einstein equations take the form:

$$
\begin{aligned}
&R_{00} = \frac{4\pi G}{c^4} g_{00}\rho, \qquad && R_{11} = -\frac{4\pi G}{c^4} g_{11}\rho, \\
&R_{22} = -\frac{4\pi G}{c^4} g_{22}\rho, \qquad && D^\mu T_{\mu\nu} = 0.
\end{aligned} \tag{8}
$$

It is then easy to verify that u and v are independent of t, and the nonzero Ricci tensors are

$$
\begin{aligned}
&R_{00} = -e^{\mu-\nu}\left[\frac{u''}{2} + \frac{u'}{r} + \frac{u'}{4}(u'-v')\right], \quad && R_{11} = \frac{u''}{2} - \frac{v'}{r} + \frac{u'}{4}(u'-v'), \\
&R_{22} = e^{-v}\left[1 - e^{v} + \frac{r}{2}(u'-v')\right]\Bigg(&& R_{33} = \sin^2\theta R_{22}.
\end{aligned} \tag{9}
$$

We derive from $D^\mu T_{\mu\nu} = 0$ that $\Gamma^0_{10} T_{00} = \frac{1}{2}u'\rho = 0$. Hence $u' = 0$. Then by (9) we have

$$
R_{00} = 0,
$$

which is a contradiction to the first equation of (8).

Central Gravitational Field

Consider a central gravitational field generated by a ball B_{r_0} with radius r_0 and mass M. In this case, the gravitational field equations becomes

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} - \nabla_{\mu\nu}\phi, \quad \text{with } \phi \text{ being a scalar function.} \tag{10}$$

It is known that the metric of the central field for $r > r_0$ can be written in the form

$$ds^2 = -e^u c^2 dt^2 + e^v dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad u = u(r), v = v(r). \tag{11}$$

Then the field equations (10) take the form

$$\begin{aligned} & v' + \frac{1}{r}(e^v - 1) = -\frac{r}{2}u'\phi', \\ & u' - \frac{1}{r}(e^v - 1) = r(\phi'' - \frac{1}{2}v'\phi'), \\ & u'' + \left(\frac{1}{2}u' + \frac{1}{r}\right)\Big(u' - v') = -\frac{2}{r}\phi'. \end{aligned} \tag{12}$$

We have derived in (Ma & Wang, 2012) an approximate gravitational force formula:

$$F = \frac{mc^2}{2} e^u \left[-\frac{1}{r}(e^v - 1) - r\phi'' \right]. \tag{13}$$

This can be further simplified as

$$\begin{aligned} & F = mMG\left(-\frac{1}{r^2} - \frac{k_0}{r} + k_1 r\right) \Bigg(\qquad \text{for } r > r_0, \\ & k_0 = 4 \times 10^{-18}\text{Km}^{-1}, \quad k_1 = 10^{-57}\text{Km}^{-3}, \end{aligned} \tag{14}$$

demonstrating the presence of both dark matter and dark energy.
Here the first term represents the Newton gravitation, the attracting second term stands for dark matter and the repelling third term is the dark energy. We note that our modified new formula is derived from first principles.

Let $x(s) \overset{\text{def}}{=} (x_1(s), x_2(s), x_3(s)) = \left(e^s u'(e^s), e^{v(e^s)} - 1, e^s \phi'(e^s)\right)$. Then the *non-autonomous* gravitational field equations (12) become an *autonomous system*:

$$\begin{aligned} x_1' &= -x_2 + 2x_3 - \frac{1}{2}x_1^2 - \frac{1}{2}x_1x_3 - \frac{1}{2}x_1x_2 - \frac{1}{4}x_1^2x_3, \\ x_2' &= -x_2 - \frac{1}{2}x_1x_3 - x_2^2 - \frac{1}{2}x_1x_2x_3, \\ x_3' &= x_1 - x_2 + x_3 - \frac{1}{2}x_2x_3 - \frac{1}{4}x_1x_3^2, \\ (x_1, x_2, x_3)(s_0) &= (\alpha_1, \alpha_2, \alpha_3) \qquad \text{with } r_0 = e^{s_0}. \end{aligned} \tag{15}$$

- The asymptotically flat space-time geometry (Minkowski) is represented by $x = 0$, which is a fixed point of the system (15).

- There is a 2D stable manifold E^s near $x = 0$, which can be parameterized by

$$x_3 = h(x_1, x_2) = -\frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{16}x_1^2 - \frac{1}{16}x_2^2 + O(|x|^3). \tag{16}$$

The field equations (15) are reduced to a 2D dynamical system:

$$\begin{aligned} x_1' &= -x_1 - \frac{1}{8}x_1^2 - \frac{1}{8}x_2^2 - \frac{3}{4}x_1x_2 + O(|x|^3), \\ x_2' &= -x_2 + \frac{1}{4}x_1^2 - x_2^2 - \frac{1}{4}x_1x_2 + O(|x|^3), \\ (x_1, x_2)(s_0) &= (\alpha_1, \alpha_2), \end{aligned} \tag{17}$$

with x_3 being slaved by (16).

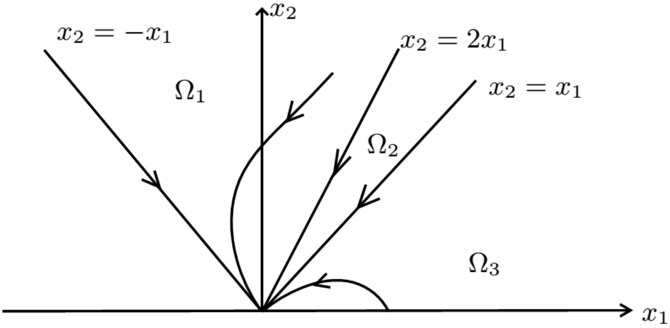

Figure: Only the orbits on Ω_1 with $x_1 > 0$ will eventually cross the x_2-axis, leading to the sign change of x_1, and to a repelling gravitational force. $x = 0$ is Minkowski, $x_1 = x_2$ is Schwarzschild.

Asymptotic Repulsion Theorem [Hernandez-Ma-Wang, 15]

For a central gravitational field, the following assertions hold true:

1) The gravitational force F is given by

$$F = -\frac{mc^2}{2} e^u u',$$

and is asymptotic zero:

$$F \to 0 \quad \text{if} \quad r \to \infty. \tag{18}$$

2) If the initial value α in (17) is near the Schwarzschild solution with $0 < \alpha_1 < \alpha_2/2$, then there exists a sufficiently large r_1 such that the gravitational force F is repulsive for $r > r_1$:

$$F > 0 \quad \text{for} \quad r > r_1. \tag{19}$$

Remarks

- The above theorem is valid provided the initial value α is small. Also, all physically meaningful central fields satisfy the condition (note that a black hole is enclosed by a huge quantity of matter with radius $r_0 \gg 2MG/c^2$). In fact, the Schwarzschild initial values are as

$$x_1(r_0) = x_2(r_0) = \frac{\delta}{1-\delta}, \qquad \delta = \frac{2MG}{c^2 r_0}. \tag{20}$$

  The δ-factors for most galaxies and clusters of galaxies are

$$\text{galaxies } \delta = 10^{-7}, \qquad \text{cluster of galaxies } \delta = 10^{-5}. \tag{21}$$

- The dark energy phenomenon is mainly evident between galaxies and between clusters of galaxies, and consequently the above theorem is valid for central gravitational fields generated by both galaxies and clusters of galaxies.
- The asymptotic repulsion of gravity (dark energy) plays the role to stabilize the large scale homogeneous structure of the Universe.

Ma-Wang, 2012

- (Einstein's PE). The space-time is a 4D Riemannian manifold $\{\mathcal{M}, g_{\mu\nu}\}$, with the metric $\{g_{\mu\nu}\}$ being the gravitational potential;
- The Einstein PGR dictates the Einstein-Hilbert action (23);
- The gravitational field equations (7) are derived using PID, and determine gravitational potential $\{g_{\mu\nu}\}$ and its dual vector field Φ_μ;

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu} - \nabla_\mu \Phi_\nu$$

- Gravity can display both attractive and repulsive effect, caused by the duality between the *attracting* gravitational field $\{g_{\mu\nu}\}$ and the *repulsive* dual vector field $\{\Phi_\mu\}$, together with their nonlinear interactions via the new gravitational field equations.
- The nonlinear interaction between $\{g_{\mu\nu}\}$ and the dual field Φ_μ gives rise to the phenomena of dark matter and dark energy:

Dark energy and dark matter are intrinsic properties of gravity.

III. A Black Hole theorem

Inside the massive ball ($r < R$), the energy-momentum tensor of an idealized fluid is in the form $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$, and $u^\mu = (1, 0, 0, 0)/\sqrt{-g_{00}}$. The 00 component of the Einstein equations is equivalent to

$$\frac{1}{r^2}\frac{dM(r)}{dr} = 4\pi\rho, \qquad M(r) \overset{\text{def}}{=} \frac{c^2 r}{2G}(1 - e^{-v}).$$

So $M(r) = \int_0^r 4\pi r^2 \rho dr$, for $0 < r < R$, is clearly the mass in the ball B_r.

Tolman-Oppenheimer-Volkoff (TOV) (1934 & 1939) metric:

$$ds^2 = -\frac{1}{4}\left(\left(1 - \frac{r^2}{R_s^2}\right)\right) c^2 dt^2 + \left(\left(1 - \frac{r^2}{R_s^2}\right)\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{22}$$

Theorem (Ma-Wang, 2014)

If the matter field in a ball B_R of radius R is spherically symmetric, and the mass M_R and the radius R satisfy

$$\frac{2GM_R}{c^2 R} = 1 \qquad \Rightarrow \qquad e^v = \left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1} \text{ is singular at } r = R$$

then the ball B_R must be a blackhole.

Black Hole Theory Based on Einstein's GR

In the exterior of the centrally symmetric massive ball B_{R_s} with total mass M, the Schwarzschild solution is

$$ds^2 = -\left[1 - R_s/r\right] c^2 dt^2 + \left[1 - R_s/r\right]^{-1} dr^2 + r^2 d\Omega^2 \qquad \text{for } r > R_s. \tag{23}$$

In the interior, the Tolman-Oppenheimer-Volkoff (TOV) metric is

$$ds^2 = -\frac{1}{4}\left[1 - \frac{r^2}{R_s^2}\right] c^2 dt^2 + \left[1 - \frac{r^2}{R_s^2}\right]^{-1} dr^2 + r^2 d\Omega^2 \qquad \text{for } r < R_s. \tag{24}$$

Both metrics have a singularity at $r = R_s$, which is called the event horizon:

$$d\tau = \left[1 - R_s/r\right]^{1/2} dt \to 0, \quad d\widetilde{r} = \left[1 - R_s/r\right]^{-1/2} dr \to \infty \text{ for } r \to R_s^+, \tag{25}$$

$$d\tau = \frac{1}{2}\left[1 - \frac{r^2}{R_s^2}\right]^{1/2} dt \to 0, \quad d\widetilde{r} = \left[1 - \frac{r^2}{R_s^2}\right]^{-1/2} dr \to \infty \text{ for } r \to R_s^-, \tag{26}$$

So time freezes at $r = R_s$: there is no motion crossing the event horizon. Namely the black hole enclosed by the event horizon $r = R_s$ is closed:

> **Nothing gets inside a black hole, and nothing gets out of the black hole either.**

1) Singularity at R_s is physical

- The Schwarzschild solution is derived from the Einstein equations under the spherical coordinate system, which has no singularity for $r > 0$. Consequently, the singularity of the Schwarzschild solution at $r = R_s$ must be intrinsic to the Einstein equations, and is not caused by the particular choice of the coordinate system. In other words, the singularity at $r = R_s$ is real and physical.

- **Mathematically forbidden coordinate transformations are used:** Classical transformations such as e.g. those by Eddington and Kruskal are **singular**, and therefore they are not valid for removing the singularity at the Schwarzschild radius. Consider for example, the Kruskal coordinates involving

$$u = t - r_*, \quad v = t + r_*, \qquad r_* = r + R_s \ln\left(\frac{r}{R_s} - 1\right)$$

This coordinate transformation is singular at $r = R_s$, since r_* becomes infinity when $r = R_s$. Namely

> *all the coordinate systems, such as the Kruskal and Eddington-Finkelstein coordinates, that are derived by singular coordinate transformations, are singular and are mathematically forbidden.*

2) Metric of a block hole

- the Schwarzschild metric is valid only outside of a black hole.
- in the interior of a black hole, the metric is the Tolman-Oppenheimer-Volkoff (TOV) metric.
- there is no singularity at the center $r = 0$.

Black Hole Theorem (Ma-Wang, J. Math. Study, 47:4(2014), 305-378)

Assume the validity of the Einstein theory of general relativity, then the following assertions hold true:

- black holes are closed: matters can neither enter nor leave their interiors;
- black holes are innate: they are neither born to explosion of cosmic objects, nor born to gravitational collapsing; and
- black holes are filled and incompressible, and if the matter field is nonhomogeneously distributed in a black hole, then there must be sub-blackholes in the interior of the black hole.

IV. Structure of the Universe

FLRW metric

Assume the validity of the cosmological principle that the Universe is homogeneous and isotropic. A. Friedmann (1922), G. Lemaître (1927), H. Robertson (1935), and A. Walker (1936) demonstrated that the Universe is either the 3D sphere S^3, or the flat $\mathbb{R}^3$, or the 3D Lobachevsky space L^3.
The FLRW metric of the spacetime is

$$ds^2 = -c^2dt^2 + R(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \tag{27}$$

where $R(t)$ is the scalar factor representing the radius of the Universe, and $k = 1, 0, -1$ stand for the sign of space scalar curvature.

$$\ddot{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)R + \frac{\Lambda c^2}{3}R, \tag{28}$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2}, \tag{29}$$

$$\dot{\rho} + 3\left(\frac{\dot{R}}{R}\right)\left(\rho + \frac{p}{c^2}\right) = 0. \tag{30}$$

Observations

- If our Universe were born to the Big-Bang, assuming at the initial stage, all energy is concentrated in a ball with radius $R_0 < R_s$, by the theory of black holes, then the energy contained in B_{R_0} must generate a black hole in $\mathbb{R}^3$ with fixed radius $R_s = 2MG/c^2$.

  In fact, according to the basic cosmological principle that the universe is homogeneous and isotropic, given the energy density $\rho_0 > 0$ of the universe, the universe will always be bounded in a black hole of open ball with the Schwarzschild radius:

$$R_s = \sqrt{\frac{3c^2}{8\pi G\rho_0}},$$

  as the mass in the ball B_{R_s} is given by $M_{R_s} = 4\pi R_s^3 \rho_0/3$. This argument also clearly shows that

  there is no unbounded universe.

- If we assume that at certain stage, the Universe were contained in ball of a radius R with $R_0 < R < R_s$, then we can prove that the Universe must contain a sub-black hole with radius r given by

$$r = \sqrt{\frac{R}{R_s}}R.$$

In fact, consider a homogeneous universe with radius $R < R_s$. Then the mass density ρ is given by

$$\rho = \frac{3M}{4\pi R^3}. \tag{31}$$

It is easy to show that the condition for a ball B_r with radius r to form a black hole is that the mass M_r in B_r satisfies that

$$\frac{M_r}{r} = \frac{c^2}{2G}. \tag{32}$$

By (31), we have

$$M_r = \frac{4\pi}{3}r^3\rho = \frac{r^3}{R^3}M.$$

Then it follows from (32) that

$$r = \sqrt{\frac{R}{R_s}}R. \tag{33}$$

Based on this property, the expansion of the Universe, with increasing R to R_s, will give rise to an infinite sequence of black holes with one embedded to another. Apparently, this scenario is clearly against the observations of our Universe, and demonstrates that our Universe cannot be originated from a Big-Bang.

Theorem (Ma-Wang, 2014)

Assume the Einstein theory of general relativity, and the cosmological principle, then the following assertions hold true:

- our Universe is not originated from a Big-Bang, and is static;
- the topological structure of our Universe is the 3D sphere S^3 such that to each observer, the corresponding equator with the observer at the center of the hemisphere can be viewed as the black hole horizon;
- the total mass $M_{\text{total}} = 3\pi M/2$ in the Universe includes both the cosmic observable mass M and the non-observable mass, regarded as dark matter, due to the space curvature energy; and
- a negative pressure is present in our Universe to balance the gravitational attracting force, and is due to the gravitational repelling force, also called dark energy.

Redshift problem

The natural and important question that one has to answer is the consistency with astronomical observations, including the cosmic edge, the flatness, the horizon, the redshift, and the cosmic microwave background (CMB) problems. These problems can now be easily understood based on the structure of the Universe and the blackhole theorem we derived.

There are three sources of redshifts: the Doppler effect, the cosmological redshift, and the gravitational redshift. Due to black hole properties of our Universe, the black hole and cosmological redshifts cannot be ignored. Due to the horizon of the sphere, for an arbitrary point in the spherical Universe, its opposite hemisphere relative to the point is regarded as a black hole. Hence, we derive the following redshift formula, which is consistent with the observed redshifts:

$$1 + z = \frac{1}{\sqrt{\alpha(r)(1 - \frac{R_s}{\widetilde{r}})}} = \frac{\sqrt{2R_s - r}}{\sqrt{\alpha(r)(R_s - r)}} \qquad \text{for } 0 < r < R_s. \qquad (34)$$

V. PID-Cosmological Model (Ma-Wang, 2015)

Metric of a homogeneous spherical universe:

$$ds^2 = -c^2dt^2 + R^2\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \tag{35}$$

where $R = R(t)$ is the cosmic radius. By (7) and with $\varphi = \phi''$, we have

$$\begin{aligned}
R'' &= -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2} + \frac{\varphi}{8\pi G}\right)R, \\
(R')^2 &= \frac{1}{3}(8\pi G\rho + \varphi)R^2 - c^2, \\
\varphi' + \frac{3R'}{R}\varphi &= -\frac{24\pi G}{c^2}\frac{R'}{R}p.
\end{aligned} \tag{36}$$

The model describing the static Universe is in the form (Ma-Wang, 2015):

$$\left\{ \begin{aligned} &\varphi = -8\pi G \left(\rho + \frac{3p}{c^2} \right), \\ &p = -\frac{c^4}{8\pi G R^2}, \\ &p = f(\rho, \varphi). \end{aligned} \right. \tag{37}$$

The negative pressure contains two parts:

$$p = -\frac{1}{3}\rho c^2 - \frac{c^2}{24\pi G}\varphi = \text{observable energy} + \text{ dark energy}. \tag{38}$$

The CMB and the Wilkinson Microwave Anisotropy Probe (WMAP) measurements manifest that the cosmic radius R is greater than the blackhole radius of the normal energy. The deficient energy is compensated by the dual gravitational potential, i.e. by the second term of (38).